A New Empirical Model for the Structural Analysis of Early-type Galaxies and a Critical Review of the Nuker Model[1]

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ABSTRACT

The *Nuker law* was designed to match the inner few ($\sim$3-10) arcseconds of predominantly nearby ($\lesssim$ 30 Mpc) early-type galaxy light-profiles; it was never intended to describe an entire profile. The Sérsic model, on the other hand, was developed to fit the entire profile; however, due to the presence of partially depleted galaxy cores the Sérsic model cannot always describe the very inner region. We have therefore developed a new empirical model consisting of an inner power-law, a transition region, and an outer Sérsic model to connect the inner and outer structure of elliptical galaxies. We have additionally explored the stability of the Nuker model parameters. Surprisingly, none are found to be stable quantities; all are shown to vary systematically with a profile's fitted radial extent, and often by more than 100%. Considering elliptical galaxies spanning a range of 7.5 magnitudes, we reveal that the central stellar densities of the underlying host galaxies *increase* with galaxy luminosity until the onset of core formation, detected only in the brightest elliptical galaxies. We suggest that the so-called "power-law" galaxies may actually be described by the Sérsic model over their entire radial range.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: fundamental parameters — galaxies: nuclei — galaxies: photometry — galaxies: structure

1. Introduction

Early *Hubble Space Telescope* (*HST*) observations of elliptical galaxies and the bulges of disk galaxies, hereafter collectively referred to as 'bulges' (Crane et al. 1993; Ferrarese et al. 1994; Jaffe et al. 1994; Forbes, Franx, & Illingworth 1995), confirmed the ground-based conclusions of Kormendy (1985) and Lauer (1985): Galaxy models with flat cores (e.g., King's 1966 model) do not describe the majority of elliptical galaxies, or at least the resolved part of the profile. In almost all galaxies surveyed, the surface brightness profile continued to rise inward until resolution was lost. Subsequently, with the enhanced image quality afforded by the *HST* have come new models to describe the centers of nearby bulges.

Ferrarese et al. (1994) introduced two classes of galaxies according to the behavior of the inner surface brightness profile. Those with a resolved core flattening towards the center were labeled as "Type I", and those that roughly follow a steep power-law all the way into the center were designated as "Type II" galaxies. These authors introduced a 4-parameter double power-law model to quantify the shape of the galaxy profile within the inner $\sim 10''$. A core radius marked the transition between the inner and outer power-laws having

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slopes β_1 and β_2, respectively. All of their "Type I" galaxies had an inner slope shallower than -0.31 (none of them had a slope of zero); all but one of their "Type II" galaxies had a slope steeper than -0.47.

Modeling a larger galaxy sample, Lauer et al. (1995; see also Kormendy et al. 1994) confirmed the above result[2] — though they interpreted it differently — and introduced a model with an additional parameter (α), which better controlled the transition between the two power-laws. This model[3] was designated the 'Nuker law' by these authors. It can be written as

$$I(r) = I_b 2^{(\beta-\gamma)/\alpha} \left(\frac{r}{r_b}\right)^{-\gamma} \left[1 + \left(\frac{r}{r_b}\right)^{\alpha}\right]^{(\gamma-\beta)/\alpha}. \tag{1}$$

The intensity at the core radius, also known as the break-radius r_b, is denoted by I_b. The inner power-law slope is now denoted by γ and the outer power-law slope is denoted by β. This model reduces to the form proposed by Ferrarese et al. (1994) when $\alpha = 2(\beta_2 - \beta_1)$. Lauer et al. (1995) refer to galaxies with $\gamma < 0.3$ as "core" galaxies and galaxies with $\gamma > 0.5$ as "power-law" galaxies.

Application of the Nuker model has proved extremely popular, and there are physical grounds to interpret the reduction in central profile slope and the implied core depletion. Many authors have discussed how the inner region of a galaxy may have been partially evacuated by the coalescence of merging supermassive black holes (SMBHs; e.g., Ebisuzaki, Makino, & Okumura 1991; Makino & Ebisuzaki 1996; Faber et al. 1997; Makino 1997; Quillen, Bower, & Stritzinger 2000; Alexander & Livio 2001; Milosavljevic & Merritt 2001). Conversely, the presence of "power-law" cusps has been used to argue for adiabatic growth of central black holes, with the growing black hole reshaping the central region (e.g., van der Marel 1999), although Ravindranath, Ho, & Filippenko (2002) have used the fitted power-law slopes to argue against this scenario. To better understand galaxy "cores", one would like to measure changes to a galaxy's inner profile relative to its original shape.

Recently, the overall "shape" of a bulge's light-profile (as parameterized by the Sérsic 1968 $r^{1/n}$ shape index n) has been shown to correlate strongly ($r_s = 0.92$) with the mass of its central SMBH (Graham et al. 2001a, 2002; Erwin, Graham, & Caon 2003). This implies a strong connection between the formation and structure of the entire bulge and the formation of the central black hole. The central regions of bulges are thus directly related to the global bulge structure, and so one would like to connect these two regimes.

The Nuker model – with 5 free parameters – can only describe the inner light-profile of a bulge; it was never designed to model an entire profile and is thus unable to make a connection between the inner profile and the overall bulge structure[4]. The Sérsic model – with 3 free parameters – matches the entire radial extent of most bulge light-profiles remarkably well, with the exception of the inner few arcseconds for some galaxies. By joining, at the break radius, an outer Sérsic profile with an inner power-law, one might hope to be able to describe the complete light-profiles of bulges when the Sérsic model alone is inadequate.

This issue will be addressed here and in a companion paper (Trujillo et al. 2003, Paper II). To do this, the merits of the individual Nuker parameters will first be explored in Section 2. Section 3 then describes the Sérsic model, which, for a number of illustrative purposes, is applied here to the central, early-type galaxy light-profiles presented in Lauer et al. (1995). Given the short-comings of both models to describe the complete light-profiles of all bulges, a new empirical model is introduced in Section 4 and is illustrated with application to both a "power-law" and a "core" galaxy profile. In Paper II, we apply the new model to radially complete profiles from a larger sample of early-type galaxies. Correlations between the global and core properties will be presented in a forthcoming paper. A recapitulation of the main points in this paper are provided in Section 5.

[2]It should be noted that Rest et al. (2001) and Ravindranath et al. (2001) have now found several high-luminosity galaxies with inner profile slope $0.3 < \gamma < 0.5$.

[3]This 5 parameter double power-law model was independently introduced by Zhao (1996) to model the spatial (i.e. not projected) density profiles of elliptical galaxies (see also Zhao 1997).

[4]A possible exception to this remark comes from the observation that the outer light-profiles of the more massive brightest cluster galaxies can be well approximated by a power-law (e.g. Graham et al. 1996). It should perhaps further be noted that the Sérsic model tends towards a power-law for large values of n.

2. The Nuker model

In those bright galaxies where Faber et al. (1997; and references therein) detected a "core", the break radius r_b and the intensity at this radius (I_b) are thought to denote the onset of a physical transition in a galaxy's profile. Together with the central velocity dispersion, Faber et al. (1997; see also Faber et al. 1987) constructed a 'core fundamental plane', from which they concluded that: cores are in dynamical equilibrium; r_b and I_b are meaningful dynamical parameters [at least in the case of "core" galaxies]; velocity anisotropy does not vary greatly among "core" galaxies; for most galaxies, the mass of any central SMBH does not dominate the core potential; and the core M/L ratio varies smoothly over the fundamental plane.

Faber et al. (1997) also noted, however, that the value of r_b [and hence I_b] is not robust for "power-law" galaxies. The difficulty the Nuker model has in obtaining stable parameters for such galaxies, and hence some quantity which reflect some fixed physical structure, is a consequence of their smooth continuously curving profiles which have no obvious core.

The Sérsic model has a smooth, continuous profile that in fact resembles the observed "power-law" profiles. The Sérsic model has also recently been shown to provide a good description to both the outer *and inner* profiles of *HST*-resolved, low-luminosity elliptical galaxies (after accounting for the central excess flux; Stiavelli et al. 2001; Graham & Guzmán 2003). Consequently, we explore the natural question: Are the so-called "power-law" galaxies simply the bright end of these Sérsic–$r^{1/n}$ galaxies without (resolved) break radii, depleted cores, or true central power-laws? To further this idea we will look at a compilation of galaxies spanning a large range in absolute magnitude.

Figure 1 shows the central surface brightnesses of the Nuker team's elliptical galaxies plotted against their absolute magnitudes. The absolute V-band magnitudes were obtained from Faber et al. (1997; their Table 2) and converted back into B-band magnitudes using their $(B-V)$ color term (their Table 1). The central surface brightnesses shown here are those of the Nuker model at $r = 0.1''$, corrected for Galactic extinction, and converted to the B-band in the same manner as done for the magnitudes[5]. Faber et al. (1997) wrote that "severe nuclei were ignored in fitting Nuker laws". This can be seen in the Nuker models fitted to both the "core" (e.g., NGC 6166 and A2052) and "power-law" galaxies in Byun et al. (1996). It is noted that the overall placement of the Nuker team's galaxies in Figure 1 is the same when the actual surface brightness within the inner $0.1''$, as given in Table 4 of Lauer et al. (1995), is used. Surface brightness values from Nuker models extrapolated to $r = 0$ were not used because of this model's power-law behavior, and hence overestimation of the true (finite) central surface brightness.

Added to this diagram are the Virgo and Fornax elliptical galaxies imaged by Caon, Capaccioli, & D'Onofrio (1993) and D'Onofrio, Capaccioli, & Caon (1994). The S0 galaxies have again been excluded as we are interested in the properties of bulges (not bulges and disks combined). The observed, model-independent central surface brightness[6] and model-independent magnitude of these elliptical galaxies are shown; there has been no recourse to the Sérsic model to determine these values, although readers are reminded that these galaxies were very well fitted with a Sérsic model by the above authors (see also D'Onofrio 2001). The dwarf elliptical galaxies in Binggeli & Jerjen (1998; their Table 1) are also included in Figure 1. These authors also avoided the nuclear point sources and so their central surface brightness measurements are from a Sérsic model fitted to the underlying bulge (extrapolated to $r = 0''$), uncontaminated by possible nuclear star clusters. Finally, the dwarf elliptical galaxies imaged with the *HST* by Stiavelli et al. (2001) are shown. The magnitudes are from their table 1, and the central surface brightness values have been read off from the Sérsic fits in their figure 1 (these readings are probably accurate to 0.05 mag arcsec^{-2}). The Sérsic fits were also made after excluding any excess central flux. Both the magnitude and central surface brightness measurements from Stiavelli et

[5]When no $(B-V)$ color term was given by Faber et al. (1997), a value of 0.9 has been adopted. This may lead to slightly increased scatter in Figure 1.

[6]It should be noted that the central surface brightnesses of these galaxies were obtained with ground-based CCD imaging under $\sim 1''$ seeing; the true central surface brightness is therefore brighter than shown here.

al.'s galaxy sample have been converted here from the V-band to the B-band assuming a constant $(B-V)$ color of 0.9.

It should also be understood that the *total*, central surface brightnesses of many galaxies included in Figure 1 are brighter than what is shown there, what is shown are estimates of the central surface brightness values of the *underlying host* galaxies. With this understanding, the central galaxy intensity is seen to increase with bulge luminosity (see also Caldwell 1983) such that $M_B \propto -(3/2)\log(I_{0,B}) = (3/5)\mu_{0,B}$ (Figure 1), but above a certain threshold ($M_B \lesssim -20.5$) one observes a reversal of this trend. The bright end of this trend can be seen in Phillips et al. (1996, their figure 6) and Faber et al. (1997, their figure 4c). *Hot galaxies have projected central stellar densities which increase with galaxy luminosity/mass until core formation occurs and a break in the light-profile is detected.* Our finding would therefore appear to disagree with the interpretation by Faber et al. (1997), who wrote, "A major conclusion is that small hot galaxies are much denser than large ones". They attributed the observed reduction in central stellar intensity as one progressed to magnitudes fainter than $M_B \sim -20.5$ as a resolution effect, using the rare 'compact elliptical' galaxy M32 to support this view.

Past modeling of M32's ground-based light-profile excluded the excess flux observed over the inner 10–15″ (e.g., Kent 1987; Choi, Guhathakurta, & Johnston 2002). If M32 was at the distance of the Virgo cluster, this central excess would show up only within the inner 1″ and the galaxy would likely be considered 'nucleated'. Excluding this unusually sharp core in M32 (Schweizer 1979 and Tonry 1984), Graham (2002a) found the underlying bulge component has an exponential-like profile; that is, it has a relatively shallow inner slope, and he derived a central surface brightness of 15.31 R-mag arcsec^{-2} for the underlying bulge. This roughly translates into a central V-band surface brightness of $\sim$15.7 mag arcsec^{-2} and places it in better agreement with the other "power-law" galaxies in Figure 4c of Faber et al. (1997). In any case, because there is evidence suggesting M32 contains an outer envelope/disk (Graham 2002a), it has not been included in our Figure 1.

It is noted that the "power-law" galaxies in Figure 1 (considered to be intermediate-luminosity elliptical galaxies, Faber et al. 1997) form a continuous extension to the dwarf (low-luminosity) elliptical galaxies which are known to be well described by the Sérsic model (Davies et al. 1988; Young & Currie et al. 1994; Jerjen, Binggeli, & Freeman 2000). Excluding the "core" galaxies, there is no apparent dE/E dichotomy in Figure 1 (see also Jerjen & Binggeli 1997). Since the Sérsic fits to the *HST* dwarf elliptical profiles of Graham & Guzmán (2003) do an excellent job of describing both the outer and the inner profiles (with a point-source used by Graham & Guzmán to fit those galaxies which are nucleated), and since Jerjen et al. (2000) were able to fit the highly resolved (due to their proximity) Milky Way and M31 dwarf spheroidal profiles at all radii with the Sérsic model, it appears that the "power-law" centers of low-luminosity ellipticals are simply the inner part of their overall Sérsic profile. Moreover, using the Sérsic-derived (finite) central surface brightness values from the Virgo and Fornax galaxies from Caon et al. (1993) and D'Onofrio et al. (1994), galaxies having $M_B \gtrsim -20.5$ overlap exactly with the Nuker team's "power-law" galaxies (Graham & Guzmán 2003). That is, galaxies having the same magnitude as the "power-law" galaxies have the same central surface brightness when derived from the inward extrapolation of the outer profile's best-fitting Sérsic model.

It therefore seems reasonable that the "power-law" galaxies may indeed simply be Sérsic–$r^{1/n}$ galaxies without cores. This is of interest because it not only helps to provide a unifying picture of galaxy structure, but reveals that the break radius for the so-called "power-law" galaxies is not something intrinsically physical to these galaxy, but is simply a parameter in a model that provides a good reproduction of the observed inner light-profile. This idea is pursued (and confirmed) by Trujillo et al. (2003; see also Figure 10 in this paper) who model the *HST* profiles of Lauer et al. (1995) and Rest et al. (2001) combined with the outer galaxy profile. That the Sérsic model can describe "power-law" galaxies over their entire radial extent, is also of interest because one replaces 5 parameters which have no clear physical meaning with 3 parameters which do, and which fit the entire profile. Furthermore, the notion that the inner regions of low-luminosity bulges should be treated differently than the outer regions (i.e., that the inner regions are described by a power-law and the outer regions by a different function) can be replaced with a single unifying model that treats both regions simultaneously.

2.1. α, β, γ, and $\gamma\prime$

The function of the parameter α in the Nuker model (Equation 1) is to allow for varying degrees of curvature in the surface brightness profile, providing a smoother, less abrupt transition between the two power-laws. While the transition region is apparently better matched in this way, a problem is known to arise when the transition region is apparently large and α becomes too small. When this occurs, the slope of the power-law components become less and less representative of the observed mean logarithmic slope on either side of the break-radius, and more representative of the slope to the extrapolated model beyond the boundaries of the fitted galaxy profile. For smaller values of α (i.e. $\lesssim$1), the presence of two power-laws often fails to emerge; instead, one continuous curving arc describes the profile. As a result, the value of the inner power-law slope (γ) is sometimes zero and/or often does not appear to reflect the observed inner slope. This aspect of the Nuker model's ability to provide an accurate quantification of the observed inner profile slope was discussed by Rest et al. (2001), who also noted the additional difficulty with the Nuker model when the break-radius is either smaller than the image resolution, or when there simply is no apparent break-radius. Therefore, in an effort to quantify the innermost resolved profile slope, they used an additional quantity to accompany the Nuker model. Rest et al. (2001) computed the negative logarithmic slope of the Nuker model at $0.1''$, which they denoted as $\gamma\prime$.

In practice, one may find that the observed inner profile is well approximated by a real power-law and the derivative at $0.1''$ matches the Nuker model parameter γ, in which case nothing is gained. Conversely, one may find that the inner profile is indeed curved (or its slope is poorly parameterized by the Nuker model due to a small value of α) and the local derivative $\gamma\prime$ does not match γ. One thus derives a new quantity ($\gamma\prime$) which is dependent on the radius where it is measured — that is, when the inner profile is not a power-law, $\gamma\prime$ is an apparent, rather than absolute, quantity. If identical galaxies are observed at different distances, then they can have different values of $\gamma\prime$ ($R = 0.1''$). This was noted by Seigar et al. (2002), but not explored or quantified.

The extent of such changes is illustrated here by computing $\gamma\prime(r = 0.1'')$ from the slope of the Nuker model (Rest et al. 2001, their equation 8) fitted to a range of $r^{1/n}$ profiles having $r_e = 10''$ and n=1, 2, 3, and 4 (see Section 3). By effectively moving each of these galaxy models three times further away[7], i.e. by simply reducing r_e by a factor of 3, $\gamma\prime$ changes from 0.00, 0.09, 0.30, and 0.51 to 0.13, 0.36, 0.63, and 0.83 for the n=1, 2, 3, and 4 models respectively. Thus, initially three galaxies would have been classified as core galaxies, but now only one would be classified as such, even though the actual galaxy structures did not change. Half of the galaxies in the sample of Rest et al. (2001) that are classified as core-galaxies using the Nuker model parameter γ are not classified as core galaxies using the derivative at $0.1''$. Some of this mismatch is probably due to the distances the galaxies are at, and hence what physical radius $\gamma\prime$ was measured at.

To conclude, γ is known to be an extrapolated quantity that does not reflect the observed inner profile slope of galaxies having small values of α. The value of $\gamma\prime$ is the slope of the profile at the innermost resolved point. It is however an *apparent* rather than an *absolute* quantity (unless, of course, the inner profile does follow a real power-law) and as such does not reflect anything intrinsic to a galaxy and should therefore not be used as such. This statement is of course also true when using the mean logarithmic slope $<\gamma>$ measured over $0.1'' < R < 0.5''$ (e.g., Lauer et al. 1995; Carollo & Stiavelli 1998). Comparisons between the value of $\gamma\prime$ (or $<\gamma>$) for different galaxies should be made with caution. For example, diagrams showing these apparent quantities versus absolute galaxy magnitude are subject to the distance effects just mentioned.

The outer power-law slope (β) of the Nuker model depends on how much of the profile's radial extent one fits; it is therefore definitely not a reliable parameter. This was recognized from the

[7]A factor of three in distance corresponds to the range of distances in the galaxy sample of Carollo & Stiavelli (1998), who computed $<\gamma>$ over 0.1–$0.5''$.

start, and Byun et al. (1996) wrote: "Even galaxies which show good agreement with the Nuker law within $10''$ in general will also fail at much larger radii beyond the field covered by the present *HST* data, as the profiles follow a curving de Vaucouleurs law, and not a power law there." It is therefore not a parameter that need be preserved in any new model that additionally fits the outer light-profile of early-type galaxies.

2.2. Robustness of the Nuker model parameters

Figure 2 shows a synthetic "core galaxy" profile. It represents a typical $r^{1/4}$ profile having an inner core. The structural parameters are such that it has an outer de Vaucouleurs profile with effective radius $r_e = 25''$, a break radius of $0.5''$ at $\mu = 14$ mag arcsec^{-2}, and an inner power-law with slope $\gamma = 0.2$. The radial extent which is fitted with the Nuker model is increased in each subsequent frame (left to right, top to bottom) in Figure 2 in order to demonstrate how the parameters of the fit change.

Not surprisingly, the value of β is strongly dependent on the fitted radial range; this was previously known, but possibly never quantified. What will be surprising to many is the unstable nature of *all* the Nuker model parameters — not just β — even when fitting a (noise- and dust-free) "core galaxy". As the fitted radial extent is increased to values typically used by Rest et al. (2001), the Nuker model break radius marches steadily outward. When the mean difference between the synthetic data and the Nuker model reaches ~ 0.03 mag arcsec^{-2} (the average value reported by the Nuker team in their fits), the derived break radius is twice the true break radius.

This effect is illustrated again with two real "core galaxy" profiles: NGC 3348 from Rest et al. (2001) and NGC 4636 from Lauer et al. (1995). These are shown in Figures 3 and 4 respectively. Exactly the same behavior as seen in Figure 2 is observed. It turns out that, due to the curvature in the profile beyond the break radius, this behavior is common to many "core galaxies" fitted with the Nuker model. Indeed, simply by looking at the published "core galaxy" profiles fitted with the Nuker model (e.g., Ravindranath et al. 2001; Laine et al. 2003), one can see for themselves how the break radii have been overestimated.

Although the covariance error analysis presented for 3 galaxies in Byun et al. (1996; their Figure 6) reveals that the 10σ χ^2 ellipses span typically $\pm 6\%$ of the fitted Nuker model break radius, we have just witnessed that such parameter coupling is not the only source of uncertainty for the Nuker model parameters. Figure 3 reveals that reducing the fitted radius by a factor of 5 — equivalent to imaging the same galaxy five times closer using the fixed Planetary Camera aperture — can change r_b (kpc) by a factor of 3. These previously unconsidered systematic errors dominate over the random errors considered in Byun et al. (1996). Additionally, because α couples with β in the Nuker model, its value is also dependent on the radial range used. This can in turn affect the value of the *inner* power-law slope γ. As a result, the Nuker model's parameters are not always robust quantities: they are sensitive to the radial region which is fitted.

3. The Sérsic model

Sérsic's (1968) $r^{1/n}$ generalization of de Vaucouleurs' (1948) $r^{1/4}$ model has proved hugely successful in describing the light-profiles of dwarf ellipticals, ordinary ellipticals, and the bulges of spiral galaxies. Early work includes that by Davies et al. (1988), Capaccioli (1989), Caon, Capaccioli, & D'Onofrio (1993, 1994), Young & Currie (1994), James (1994), and Andredakis, Peletier, & Balcells (1995).

Recently, Graham, Trujillo & Caon (2001b; see also Graham 2002b) showed a strong correlation ($r > 0.8$, significance $> 99.99\%$) exists between the Sérsic shape parameter n and literature velocity dispersion measurements for those early-type galaxies studied by Caon et al. (1993) and D'Onofrio et al. (1994). Central stellar velocity dispersions are, of course, completely independent from estimates of n obtained from the galaxy light-profile. This clearly shows that the Sérsic index n is not simply an extra parameter added to improve the fits of bulge light-profiles, but that it traces real physical differences in galaxies. A number of authors have suggested that the different profile shapes are connected to the gravitational potentials and total masses of the bulges (e.g., Caon et al. 1993; Andredakis et al. 1995; Hjorth & Madsen 1995; Seigar & James 1998; Márquez et al. 2000, 2001; Trujillo, Graham, & Caon 2001).

The radial intensity distribution of the Sérsic model is given by the expression

$$I(r) = I_e \exp\left\{-b_n\left[\left(\frac{r}{r_e}\right)^{1/n} - 1\right]\right\}, \qquad (2)$$

where I_e is the intensity at the half-light radius r_e. The quantity b_n is a function of the shape parameter n, and is defined so that r_e is the radius enclosing half the light of the galaxy model; it can be approximated by $b_n \approx 1.9992n - 0.3271$, for $1 \lesssim n \lesssim 10$ (see, e.g., Caon et al. 1993; Graham 2001). Figure 5 reveals the behavior of the $r^{1/n}$ model for values of n ranging from 1 to 10; $n = 4$ reproduces the de Vaucouleurs model, while $n = 1$ reproduces an exponential profile.

It is clear from Figure 5 that profiles with low values of n (which observations show us are the low-luminosity bulges, e.g., Graham et al. 1996, their figure 11) have "cores" (using the definition $\gamma < 0.3$), while profiles with large values of n (the brighter bulges) would be described as "power-law" galaxies. However, this is exactly the opposite of what Faber et al. (1997) found (e.g., their Figure 4; see also Figure 7 of Rest et al. 2001): cores are found in the *brighter* bulges, while it is the relatively fainter bulges which have power-law centers. To avoid confusion, it is important to distinguish between what might be called 'apparent cores' from low n galaxies and cores which have possibly been created by supermassive black holes in high-luminosity (high n) galaxies. The former should perhaps not even be referred to as "cores" at all because they do not represent any departure from the inward extrapolation of the outer galaxy profile. Because there were very few low-luminosity galaxies (with probable Sérsic indexes $\lesssim 3$) in the Lauer et al. (1995) sample, the ambiguity between "apparent" and "real" cores did not become an issue. Studies of lower luminosity ellipticals and spiral galaxy bulges are however more problematic.

Figure 5 also reveals that the inner profile slopes, when measured over the same radial range (in terms of fraction of the effective radius), should be equal for any sample of bulges with the *same* Sérsic shape. If, however, one used a fixed *angular* range (e.g., in arcseconds), for a sample of galaxies at a range of distances (and/or with intrinsically different scale-lengths), then one will obtain a range of different inner profile slopes, even if the galaxies all have the same structural shape (as illustrated in Section 2.1).

For the Sérsic model, it is simple to show that

$$\gamma\prime(r\prime) \equiv -d\log I(r\prime)/d\log r \qquad (3)$$

is equal to

$$(b_n/n)(r\prime/r_e)^{1/n}. \qquad (4)$$

This can be approximated by $2(r\prime/r_e)^{1/n}$. Thus, at constant $(r\prime/r_e)$, $\gamma\prime$ is a monotonically increasing function of the Sérsic index n. Solutions to $\gamma\prime$ are shown in Figure 6.

The value of n is well known to increase with bulge luminosity, and n is consequently a function of position along the L-shaped trend seen in Figure 1. Given the correlation between n and $\gamma\prime$ in Figure 6, one would expect to see $\gamma\prime$ increase with bulge magnitude until a core starts to appear at the higher luminosity end[8]. This is indeed what is found in Graham & Guzmán (2003; their Fig.8).

It was suggested in Section 2 that the so-called "power-law" galaxies are actually Sérsic–$r^{1/n}$ galaxies. To explore how the Nuker model can imitate a pure Sérsic profile when fitted to a restricted radial range, Figure 7 displays the results of fitting Nuker models to four $r^{1/n}$ models with values of n equal to 1, 2, 3, and 4. The fitting has been done in such a way as to try and match the break-radius to the radius where the change in the logarithmic profile slope is observed to be a maximum. What this means is that the fitting routine was prevented from setting the break-radius to infinity[9] (although, given the results in the literature, r_b never tends to ∞ — possibly because of restrictions placed in the codes to keep r_b bound). One can clearly see that fitting a 5-parameter function (the Nuker model) to a limited radial extent of a 3-parameter function (the Sérsic

[8]The Sérsic model is known to fit the bulk of a bulge's light-profile, but for large values of n, its rising inner profile cannot describe the presence of cores in large luminous elliptical galaxies (Kormendy 1985; Lauer 1985). From Figure 1 and Figure 6 it is predicted that a better sampling of galaxies with $M_B \sim -17$ mag and Sérsic indexes $1.5 \lesssim n \lesssim 3$ (i.e. the brighter dwarf elliptical galaxies) should reveal more galaxies with $0.3 < \gamma\prime < 0.5$. However, there is the issue of galaxy distance.

[9]The Nuker model is equivalent to the $r^{1/n}$ model when $r_b \to \infty$, $\gamma = 0$, and $\alpha = 1/n$ (Byun et al. 1996).

model) can result in what many would consider a satisfactory fit.

3.1. Modeling the Nuker profiles with a Sérsic model

Major-axis surface brightness profiles for 42 predominantly early-type galaxies imaged with the *HST* are given in tabular form in Lauer et al. (1995). A logarithmically spaced sampling of the light-profiles was used, providing greater detail in the central regions. These profiles were extracted from pre-refurbishment *HST* Planetary Camera images (taken with the F555W filter) and then deconvolved to account for the effects of spherical aberration (see Lauer et al. 1995 for details). Nuker models are fitted in Byun et al. (1996)[10].

Figure 8 shows the results of fitting the 3-parameter Sérsic model to the first eight NGC galaxies in the sample of Lauer et al. (1995). Figures for the remaining galaxies show the same behavior and are therefore not shown here. The data within the inner $0.13''$, the radius inside of which the profiles were deemed unreliable by Lauer et al. (1995), were not included in the fitting routine. Quite clearly, the inner $\sim 10''$ of some galaxies are very well modeled with the 3-parameter Sérsic model. This is true for galaxies labeled by the Nuker team as either "power-law" galaxies (e.g., NGC 1023, NGC 1172) or "core" galaxies (e.g., NGC 720, NGC 1399). Relative to the Sérsic model, NGC 1331 displays evidence for a large excess flux within the inner $0.3''$. On the other hand, NGC 1400 appears to have a small core within the inner $\sim 0.2''$.

Despite the results of Figure 8, we are not trying to argue that one should fit the inner $\sim 10''$ of an appreciably larger profile with a Sérsic model. Instead, we want to demonstrate the inadequacy of the inner $\sim 10''$ for drawing reliable conclusions unless one has further information. For example, with only knowledge of the inner profile, is there really an evacuated core in NGC 720? Fitting a Sérsic model one would say no, fitting a Nuker model one would say yes.

In the following section we will advocate a new definition for a core, specifically, a deficit of central starlight (not due to dust) relative to the inward extrapolation of the outer light-profile. This differs from the Nuker team's definition which is dependent on the inner profile slope.

[10]Byun et al. (1996) fitted the Nuker model to the *mean* profiles, rather than the major-axis profiles given in Lauer et al. (1995).

4. A new empirical light-profile model

We have discussed how fitting a Nuker model could lead to a false conclusion regarding the existence of a (partially evacuated) core. Additionally, we have just seen in Figure 8 that if one only has the inner portion of a larger profile, then a Sérsic model is also capable of fitting the data — even when a real core may be present. One obvious way to avoid this potential confusion, and at the same time enable one to connect the inner and outer galaxy structure, is to increase the radial extent of the galaxy's surface brightness profile one is investigating. Given that galaxies with evacuated cores probably do exist, we need a new model to describe the entire radial extent of a profile. Of course, the additional data points in the outer profile provide more information than contained in the inner few arcseconds and therefore enable one to determine an additional parameter beyond the capabilities of the Nuker model. Specifically, one can fit for the curvature in the outer profile where a power-law is known to be inadequate for the majority of galaxies.

Modifying the Sérsic model through the inclusion of an inner power-law, or similarly, modifying the Nuker model through the transformation of the outer power-law to a Sérsic function, one obtains the expression

$$I(r) = I' \left[1 + \left(\frac{r_b}{r}\right)^{\alpha}\right]^{\gamma/\alpha} \exp\left\{-b_n[(r^{\alpha} + r_b^{\alpha})/r_e^{\alpha}]^{1/(\alpha n)}\right\}, \tag{5}$$

where r_b is the break-radius separating the inner power-law having logarithmic slope γ from the outer Sérsic function having a shape parameter n and effective half-light radius r_e. The quantity b_n is a function of n and has the usual meaning (see section 3). By leaving b_n defined this way, the value of r_e is the effective half-light radius of the outer $r^{1/n}$ profile beyond the transition region, and not the half-light radius of the new model. The effective surface brightness of the outer Sérsic profile is obtained by putting $r = r_e$ and $r_b = 0$ in equation 5, while r_b retains its value in equation 6 below. The intensity I_b at the break-radius r_b can be evaluated from the expression

$$I' = I_b 2^{-(\gamma/\alpha)} \exp\left[b_n (2^{1/\alpha} r_b/r_e)^{1/n}\right]. \qquad (6)$$

The final parameter, α, controls the sharpness of the transition between the inner (power-law) and outer (Sérsic) regimes — higher values of α indicating sharper transitions. It can be held fixed (for example, $\alpha = 100$ is a good approximation to a perfectly sharp transition), or it can be varied if one is interested in accurately matching the transition region. In practice (see Trujillo et al. 2003), we find that a sharp transition ($\alpha \gtrsim 3$) may be preferable; high values of α also minimize possible coupling of the parameters, which may compromise the inner profile slope (as can happen with Nuker model).

In spite of the previously discussed issues associated with the inner power-law slope of the Nuker model (section 2.1), γ has remained a quantity of interest in the literature. Even with a sharp transition (i.e. large values of α) our new empirical model has curvature built into it through the parameter n. Consequently, the problems that the Nuker model had in trying to accommodate curvature (via small values of α) should not plague this new model, and values of γ derived from fitting equation 5 should more accurately reflect the observed inner profile slope.

Despite appearances, equation 5 is remarkably simple. When r is less than r_b equation 5 approximates a simple power-law with slope γ; when r is greater than r_b equation 5 represents a Sérsic model. There is however a variable transition region, as mentioned above, which depends on the value of α. Setting r_b and γ equal to zero, one recovers a pure $r^{1/n}$ model at all radii. Additionally, when $n \rightarrow \infty$ it can be shown that equation 5 approximates two power-laws separated at r_b. Figure 9 shows equation 5 for different sets of parameters for this new model.

Figure 10 presents two examples where the new empirical model has been applied to real galaxies. One can see that the "power-law" galaxy NGC 5831 from Rest et al. (2001) is well described by a Sérsic model over its entire observed radial range ($0.1''$ to $\sim 3r_e$). On the other hand, NGC 3348 from Rest et al. (2001) clearly displays a flattening of the inner profile relative to the outer Sérsic profile; this break is well matched by the new model.[11] In Trujillo et al. (2003), we present these and similar fits to approximately twenty bona fide elliptical galaxies. Using archival *HST* data matching that of the inner profiles from Lauer et al. (1995) or Rest et al. (2001), we verify that "power-law" galaxies have pure Sérsic profiles, while core galaxies are well fit by our new model.

We have explored the stability of the parameters in the new model and found that in order to obtain a robust estimate of the curvature in the outer profile, one requires a profile which extends to, typically, at least the half-light galaxy radius. Although, we would advocate that one should fit as much of the profile as possible, preferably out to 2–3 r_e. As our main drive has been the issue of recovering core sizes, we remark here on our fits to the "core" galaxy NGC 3348. Fitting equation 5 to our full $70''$ profile (Figure 10), we obtained a break radius of $0.45''$, $\mu_b = 15.31$ mag arcsec^{-2}, $\gamma = 0.18$, r_e=21.82$''$, and $n = 3.87$. Truncating the profile one data point at a time, these numbers changed by at most 5%. Upon reaching a profile with a radial extent of $21.5''$ ($\sim 1r_e$), application of equation 5 gave $r_b = 0.45''$, $\mu_b = 15.31$ mag arcsec^{-2}, $\gamma = 0.18$, $r_e = 20.84''$, and $n = 3.81$, with an r.m.s. scatter of 0.028 mag for the fit. Thus, use of the complete galaxy light-profile (or at least enough of the profile to reliably determine the curvature beyond the break radius) can enable one to make robust estimates of the core size. Doing this, the core size we obtained for NGC 3348 is more than a factor of two smaller than the value published in Rest et al. (2001). The reason for this is apparent in Figures 2 – 4. Fitting the full $55''$ profile of NGC 5831 yielded $r_e = 28.28''$ and $n = 4.91$; fitting to only $27.3''$ ($\sim 1r_e$) gave $r_e = 27.00$ and $n = 4.83$, within 5% of each other.

Lauer et al. defined a "core" to be "the region interior to a sharp turndown or break in the steep outer brightness profile, provided that the profile interior to the break has $\gamma < 0.3$." We suggest an alternative definition, such that a core refers to a deficit (not due to dust) of flux relative to the inward extrapolation of the outer Sérsic profile. A "core" would thus refer to something which is

[11]NGC 4636 (Fig. 4) is not shown because given its size of 6×4.7 arcminutes, we have no outer light-profile in the *HST* image.

likely to be real, rather than an apparent feature which can appear in plots of μ versus $\log r$ (see Figure 7).

Consideration of this new model (equation 5) is also suggested for studies requiring a realistic gravitational lens model for elliptical galaxies at intermediate redshifts (e.g., Evans & Wilkinson 1998; Muñoz, Kochanek, & Keeton 2001; Chae 2002; Keeton 2002). Other interesting areas of research are the feeding of SMBHs via the capture of stars and dark matter (e.g. Zhao, Haehnelt, & Rees 2002) and the evaluation of the central mass deficit possibly excavated by coalescing massive black holes (Ebisuzaki et al. 1991; Faber et al. 1997; Milosavljevic & Merritt 2001; Milosavljevic et al. 2002: Ravindranath, Ho, & Filippenko 2002; Komossa et al. 2003). Current research assumes that galaxies initially had the same inner profile[12] (such as an $r^{1/4}$ profile or a density profile where $\rho(r) \sim r^{-2}$, e.g. Volonteri, Haardt, & Madau 2003) and compares this with how the profile actually looks. It might be of interest to replace this assumption with the inward extrapolation of the observed outer Sérsic profile. Bulges *are not* structurally homologous systems; their properties vary systematically with total luminosity and mass. To assume structural homology very likely introduces a systematic bias into these types of analysis. Lastly, Ravindranath et al. (2002) have used the break radii from the Nuker model to estimate the evacuated core masses in their sample of galaxies (Ravindranath et al. 2001) and those from Rest et al. (2001). This may explain why they found a weaker trend than expected, and one with considerable scatter, between the central SMBH mass and the ejected mass. This issue will be explored in Trujillo et al. (2003, in prep).

5. Summary

None of the 5 parameters of the Nuker model are found to be robust. Hence, they can not represent any fixed physical quantity. Recent methods which have tried to circumvent one of the Nuker parameters by measuring the inner power-law slope at some fixed radius in arcseconds ($\gamma\prime$), but without taking galaxy distance or size into account, are shown (quantitatively) to be subject to strong biases. Measured values of $\gamma\prime$ (and $<\gamma>$) are not intrinsic to a galaxy, and can change considerably if the same galaxies are located at different distances without any actual change to the intrinsic galaxy structure.

As observed by previous authors (e.g. Caldwell 1983; Jerjen & Binggeli 1997) dwarf elliptical galaxies form a continuous extension to the intermediate luminosity elliptical galaxies in the central surface brightness — absolute magnitude plane (Figure 1), such that $M_B \propto (3/5)\mu_{0,B}$. Faber et al. (1997) wrote, "A major conclusion is that small hot galaxies are much denser than large ones" and that "the apparent turndown in [central] surface brightness at faint magnitudes ... is probably a resolution effect". However, observations of the *underlying host galaxy* (i.e., excluding nuclear sources) reveal, over a range of 7.5 mag, that small hot galaxies are actually less centrally dense than larger hot galaxies - this is not an artifact of resolution. We instead attribute the observed turndown in central surface brightness at *bright* magnitudes ($M_B \lesssim -20.5$ mag) to the presence of galactic cores. Lastly, it is noted that the lower luminosity elliptical galaxies are known to be well described by the Sérsic model, and it is suggested here that the "power-law" galaxies may in fact simply be Sérsic–$r^{1/n}$ galaxies with no resolved core. The 3-parameter Sérsic model provides a remarkably good fit to the 42 inner galaxy light-profiles initially studied by the Nuker team. In some cases the quality of the fit may be because the so-called "power-law" galaxies are Sérsic–$r^{1/n}$ galaxies all the way into the resolution limit, while in other cases it is probably a result of the limited radial extent of the profiles.

Whether or not a "core" represents a real physical change, or just an apparent change in profile slope, can be determined by looking at the entire light-profile, rather than just the central profile, and we advocate a new definition for a "core" as a deficit in central flux relative to the outer Sérsic profile. The results of doing this are shown in Trujillo et al. (2003) for a sample of bonafide elliptical galaxies. In order to model the entire light-profiles, a new empirical model comprised of an outer Sérsic function and an inner power-law has

[12]Current assumptions that faint elliptical galaxies (the assumed building blocks of core galaxies) have isothermal cusps with $\rho \sim r^{-2}$ may not be correct. Low-luminosity ellipticals have small values of n and therefore shallow inner cusps (Figure 5, see also Graham & Guzmán 2003), although they may possess additional central components.

been developed and is presented here and will be described in greater mathematical detail in Trujillo et al. (2003).

By combining an inner power-law with an outer Sérsic profile, one should be better able to:

1. Explore where and how the $r^{1/n}$ model fails to provide a good match to the inner light-profile, and thereby test whether the so-called "power-law" galaxies are actually galaxies described by an $r^{1/n}$ model down to the resolution limit (i.e. having no resolvable cores, and not having an inner power-law profile).
2. Quantify central excess fluxes known to exist in many galaxies.
3. Search for connections between the shape of a galaxy's outer profile, as represented by n, and the properties of its core.
4. Quantify the slope and break-radii of "cores" normalized to the galaxy's effective half-light radius.
5. Model the gravitational lensing deflection caused by distant elliptical galaxies.
6. Search for correlations between supermassive black hole mass (possibly derived from the $\log n$–$\log M_{\rm bh}$ relation, Graham et al. 2001a, 2003, Erwin et al. 2003), break-radii, and the central flux/mass deficit in galaxies having partially evacuated cores.

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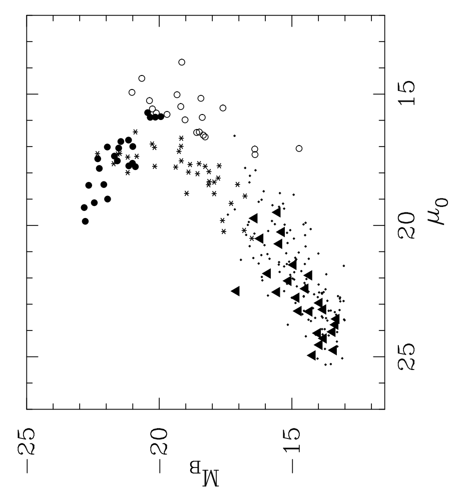

Fig. 1.— Absolute B-band galaxy magnitude M_B versus central B-band surface brightness μ_0 for the entire gamut of elliptical galaxies. The dwarf elliptical galaxies from Binggeli & Jerjen (1998; see also Jerjen & Binggeli 1997) are points, while the dwarf elliptical galaxies from Stiavelli et al. (2001) are triangles. The intermediate luminosity Virgo and Fornax elliptical galaxies from Caon et al. (1993) and D'Onofrio et al. (1994) are given as stars, the so-called "power-law" galaxies from Faber et al. (1997) are open circles, while their "core" galaxies are filled circles. The outlying dwarf elliptical galaxy from Stiavelli et al. (2001) is VCC9, and the outlying "power-law" galaxy is V1627. H_0 = 70 km s^{-1} Mpc^{-1} was used, as was a Virgo distance modulus of 31.2.

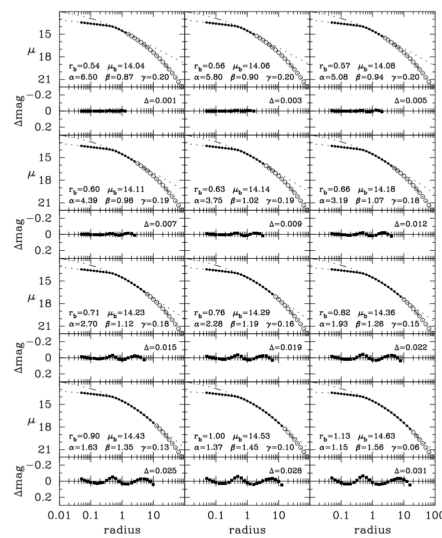

Fig. 2.— A model "core" galaxy, along with Nuker-model fits using progressively larger radial ranges. The best-fitting Nuker models (dotted lines) have been derived from modeling a profile consisting of an outer de Vaucouleurs profile (dash-dot-dot-dot curve) having $r_e = 25''$ and an inner power-law with a negative logarithmic slope of 0.2. The break radius of this model, delineated by the filled and open circles, is at $0.5''$ and μ_b=14.0 mag arcsec^{-2}, and a value of α = 8.0 (see equation 5) has been used. The radial extent of the fitted data (filled circles) is increased from left to right and top to bottom. Although every fit looks acceptable, the actual Nuker model parameters (inset in figure) can be seen to vary systematically. The r.m.s. scatter Δ mag for each fit is given in each panel.

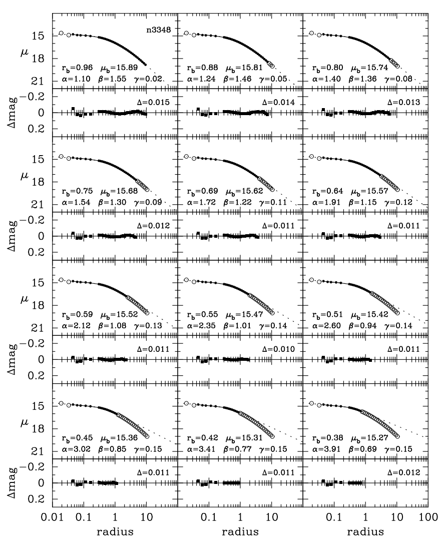

Fig. 3.— Nuker-model fits (solid and dotted lines), using progressively smaller radial ranges, are applied to the major-axis F702W surface-brightness profile of NGC 3348 (from Rest et al. 2001). The radial extent of the fitted data (filled circles) decreases from left to right and top to bottom. Although every fit looks acceptable, as we saw in figure 2 the actual Nuker model parameters (inset in figure) vary systematically with the fitted radial extent (c.f Fig. 2. Rest et al. (2001) reported a break radius $r_b = 0.99''$ for this galaxy, based on their Nuker-model fit.

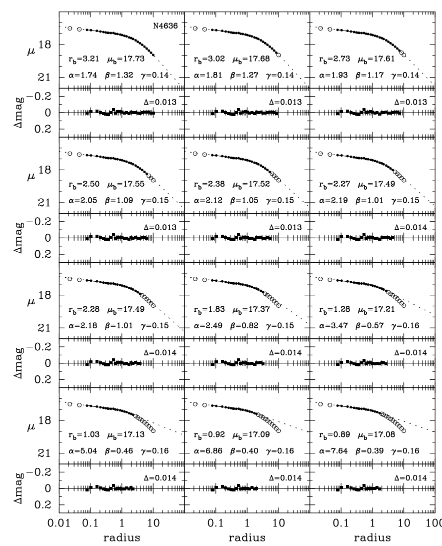

Fig. 4.— Same as Figure 3, but using the mean-axis F555W galaxy profile of NGC 4636 from Lauer et al. (1995). Byun et al. (1996) reported a break radius of $3.21''$ for this galaxy. If this galaxy had of been five times closer, a radius five times smaller (in physical units) would have been sampled with a $10''$ profile, and a break radii some 3 times smaller may have been reported.

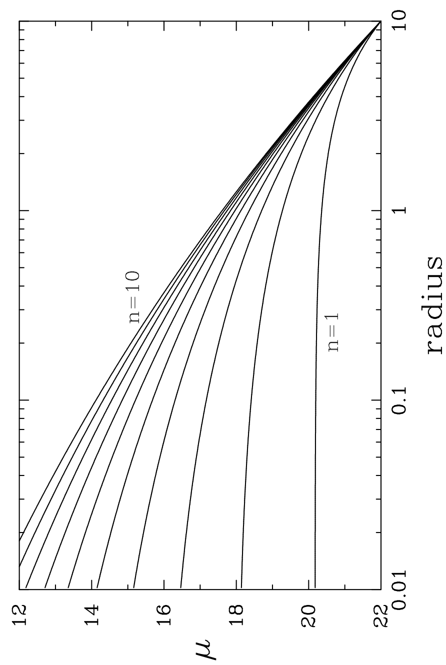

Fig. 5.— Sérsic profiles with values of n ranging from 1 to 10 are shown on a logarithmic scale. The effective half-light radius (r_e) is equal to 10 for each profile.

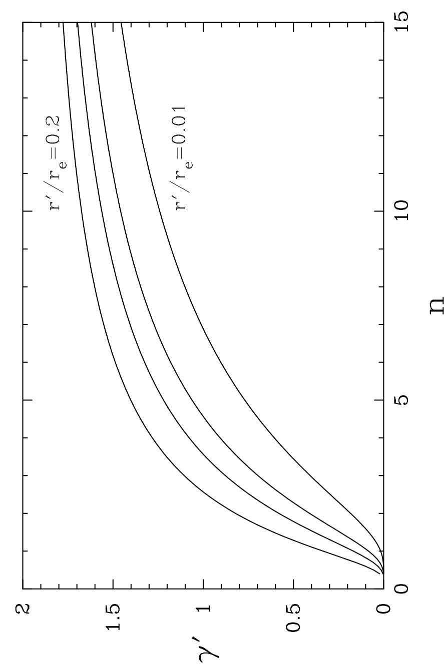

Fig. 6.— The negative logarithmic slope of the Sérsic model ($\gamma\prime$), at different fractions of the half-light radius r_e (0.01, 0.05, 0.1, 0.2), is shown as a function of the Sérsic index n.

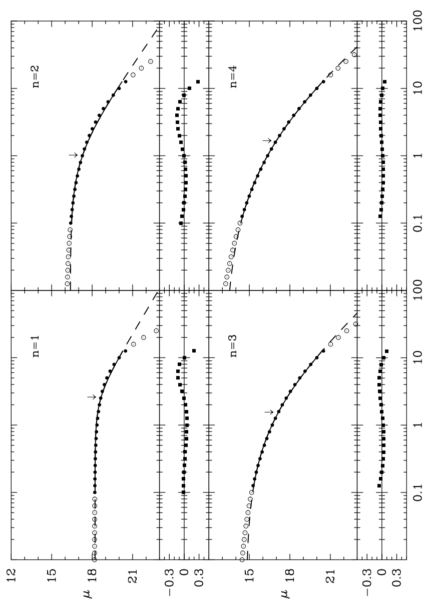

Fig. 7.— Four Sérsic profiles (small circles) with n=1,2,3, and 4, $r_e = 10''$, and $\mu_e = 20$ mag arcsec^{-2}. The solid lines are Nuker model fits to the data points (filled circles); the dashed lines are extrapolations outside the region of the fit. The Nuker model break-radii are indicated with arrows.

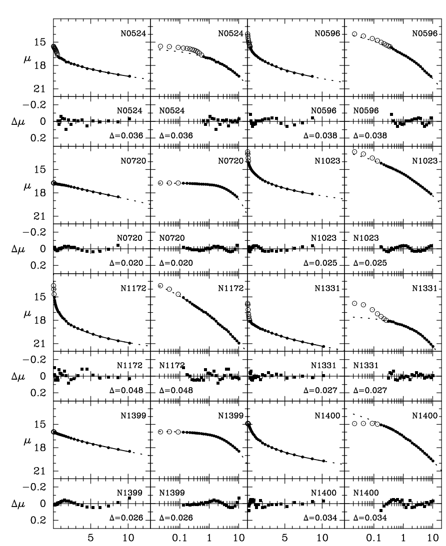

Fig. 8.— Sérsic fits to the first eight NGC galaxy profiles published in Lauer et al. (1995). Dots indicate data points used in the fit, while open circles indicate points not used for the fit (c.f. Byun et al. 1996, their Fig.3). The radius is in arcseconds.

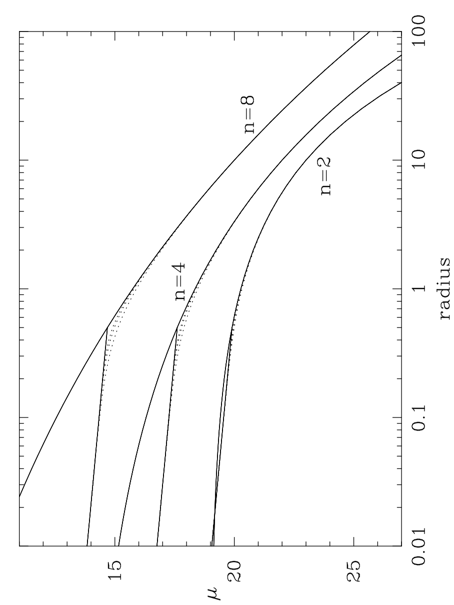

Fig. 9.— The new empirical model given in Equation 5 is illustrated by the dotted curves for a range of structural parameters. Profiles with values of α equal to 2, 3, and 4 are shown, the latter giving the sharpest transition. In all models $r_e = 10$, $r_b = 0.5$, and $\gamma = 0.2$. For comparison, an inner power-law with slope equal to -0.2 is shown (solid line) as are Sérsic profiles (solid curves) having the same Sérsic shape index n as the new empirical model (values of n are given in the Figure).

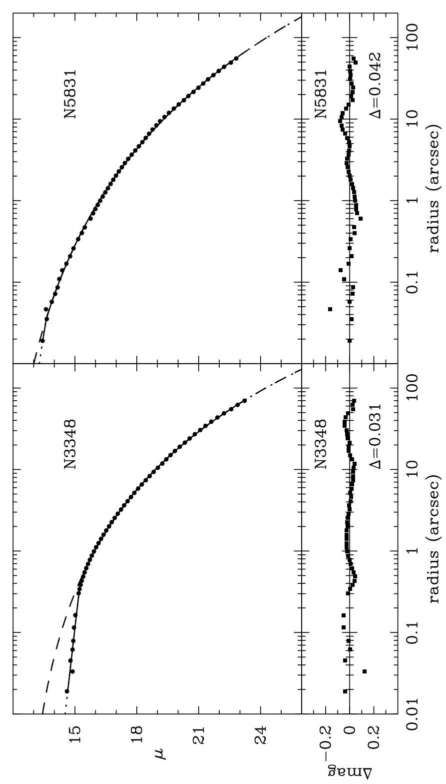

Fig. 10.— Two *HST* major-axis surface-brightness profiles of elliptical galaxies. The inner $r \lesssim 15''$ are the (deconvolved) profiles from Rest et al. (2001), while the outer points are from the full WFPC2 mosaic of the same exposures (see Trujillo et al. 2003 for full details). The solid lines are fits using the new empirical model of equation 5, with inner and outer extrapolations indicated by the dotted lines. Extrapolations of the outer, Sérsic-like part of the model inward, past the break radius, are indicated by the dashed lines. For NGC 5831, a "power-law" galaxy according to Rest et al. (2001) with $r_b = 1.78''$, our best fit is essentially a pure Sérsic model. In NGC 3348, which Rest et al. (2001) classified as a "core" galaxy with break radius $r_b = 0.99''$, there is a clear inner break from the Sérsic profile; we find $r_b = 0.45''$. The r.m.s. scatter Δ mag for the fit is given in the plot.